

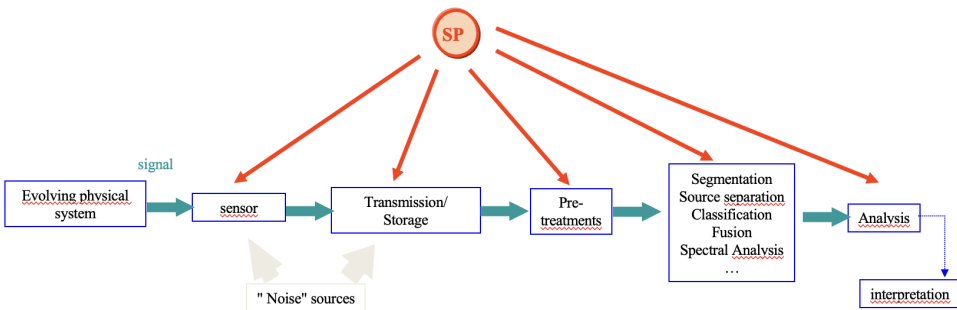
GdR IASIS

Information, Apprentissage, Signal, Image, vISion

Journée Optimisation
Sciences Informatiques, 4 octobre 2024

- **Members:** 4000 (200 laboratories, 20 members of the club of industrial partners or EPICs)

- **Members:** 4000 (200 laboratories, 20 members of the club of industrial partners or EPICs)

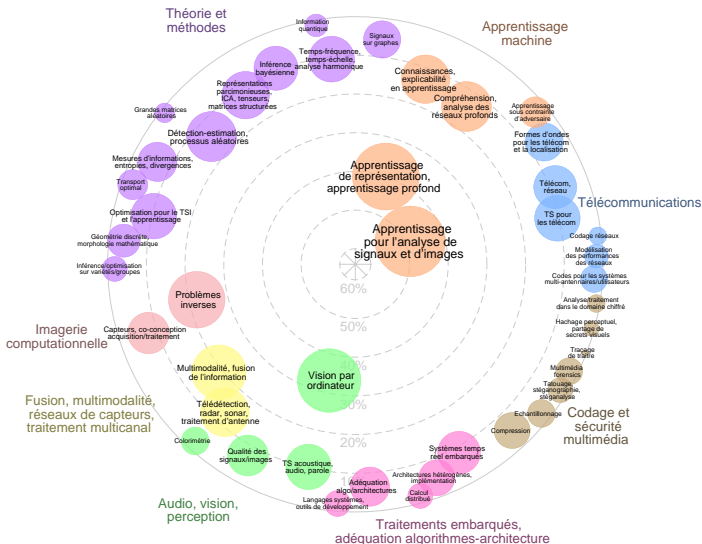


- **Members:** 4000 (200 laboratories, 20 members of the club of industrial partners or EPICs)

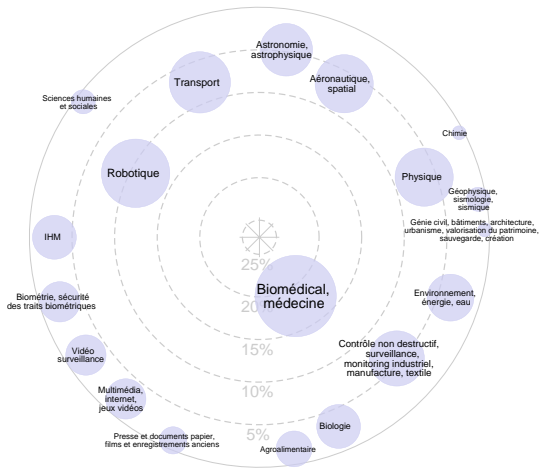
Eight axes:

- 1- Machine learning
- 2- Theory and methods
- 3- Computational imaging
- 4- Fusion, multimodality, sensor networks, multicanal processing
- 5- Audio, vision, perception
- 6- Algorithm-architecture adequacy, embedded processing
- 7- Multimedia coding and security
- 8- Telecommunications

"Thematic" activities of the GdR by keywords



"Applicative" activities of the GdR by keywords



Theory and Methods: some scientific days

1. Signal processing on graphs
2. Covariance matrices in statistics and learning
3. Multivariate polynomials in statistics and signal processing
4. Simulation and optimization
5. Optimization
6. Classical and quantum information measures (entropy,...)
7. Robust statistics: recent developments
8. Constrained matrix factorization
9. Evaluation of optimization algorithms and Monte Carlo algorithms (benchmarks)
10. Exact L0 optimization

Theory and Methods: some scientific days

1. Signal processing on graphs
2. Covariance matrices in statistics and learning
3. Multivariate polynomials in statistics and signal processing
4. Simulation and **optimization**
5. **Optimization**
6. Classical and quantum information measures (entropy,...)
7. Robust statistics: recent developments
8. Constrained matrix factorization
9. Evaluation of **optimization** algorithms and Monte Carlo algorithms (benchmarks)
10. Exact L0 **optimization**

Theory and Methods: some scientific days

1. **Signal processing on graphs**
2. Covariance matrices in statistics and **learning**
3. Multivariate polynomials in statistics and signal processing
4. **Simulation and optimization**
5. **Optimization**
6. Classical and quantum information measures (entropy,...)
7. **Robust statistics: recent developments**
8. **Constrained matrix factorization**
9. **Evaluation of optimization algorithms and Monte Carlo algorithms (benchmarks)**
10. **Exact L0 optimization**

Computational imaging

1. Inverse models in astrophysics
2. Non-conventional optical imaging
3. Co-design (hybrid sensors and algorithms for innovative systems)
4. Inverse problems for tomography
5. Radio astronomy

Computational imaging

1. **Inverse models in astrophysics**
2. **Non-conventional optical imaging**
3. **Co-design (hybrid sensors and algorithms for innovative systems)**
4. **Inverse problems for tomography**
5. **Radio astronomy**

Typical problems (non exhaustive)

- **Optimization in machine learning** methods for estimating network parameters, hyper-parameters...

Typical problems (non exhaustive)

- **Optimization in machine learning** methods for estimating network parameters, hyper-parameters...
- **Inverse problems** in image processing (denoising, restoration, 3D reconstruction, image registration, motion estimation,...)

Typical problems (non exhaustive)

- **Optimization in machine learning** methods for estimating network parameters, hyper-parameters...
- **Inverse problems** in image processing (denoising, restoration, 3D reconstruction, image registration, motion estimation,...)
- **Image compression**: finding the smallest bit rate while keeping the best quality

Typical problems (non exhaustive)

- **Optimization in machine learning** methods for estimating network parameters, hyper-parameters...
- **Inverse problems** in image processing (denoising, restoration, 3D reconstruction, image registration, motion estimation,...)
- **Image compression**: finding the smallest bit rate while keeping the best quality
- **Signal transmission**: maximizing spectral efficiency as a function of channel propagation conditions

Typical problems (non exhaustive)

- **Optimization in machine learning** methods for estimating network parameters, hyper-parameters...
- **Inverse problems** in image processing (denoising, restoration, 3D reconstruction, image registration, motion estimation,...)
- **Image compression**: finding the smallest bit rate while keeping the best quality
- **Signal transmission**: maximizing spectral efficiency as a function of channel propagation conditions
- **Wireless communications**: Bandwidth allocation

Typical problems (non exhaustive)

- **Optimization in machine learning** methods for estimating network parameters, hyper-parameters...
- **Inverse problems** in image processing (denoising, restoration, 3D reconstruction, image registration, motion estimation,...)
- **Image compression**: finding the smallest bit rate while keeping the best quality
- **Signal transmission**: maximizing spectral efficiency as a function of channel propagation conditions
- **Wireless communications**: Bandwidth allocation
- **Image segmentation** by functional optimization to find contours, homogeneous areas, (e.g. Mumford Shah functional)

Typical problems (non exhaustive)

- **Optimization in machine learning** methods for estimating network parameters, hyper-parameters...
- **Inverse problems** in image processing (denoising, restoration, 3D reconstruction, image registration, motion estimation,...)
- **Image compression**: finding the smallest bit rate while keeping the best quality
- **Signal transmission**: maximizing spectral efficiency as a function of channel propagation conditions
- **Wireless communications**: Bandwidth allocation
- **Image segmentation** by functional optimization to find contours, homogeneous areas, (e.g. Mumford Shah functional)
- **Classification** (SVM, Random Forest, Neural Networks,...)

Typical problems (non exhaustive)

- **Optimization in machine learning** methods for estimating network parameters, hyper-parameters...
- **Inverse problems** in image processing (denoising, restoration, 3D reconstruction, image registration, motion estimation,...)
- **Image compression**: finding the smallest bit rate while keeping the best quality
- **Signal transmission**: maximizing spectral efficiency as a function of channel propagation conditions
- **Wireless communications**: Bandwidth allocation
- **Image segmentation** by functional optimization to find contours, homogeneous areas, (e.g. Mumford Shah functional)
- **Classification** (SVM, Random Forest, Neural Networks,...)
- **Fusion** of multimodal data

Typical problems (non exhaustive)

- **Optimization in machine learning** methods for estimating network parameters, hyper-parameters...
- **Inverse problems** in image processing (denoising, restoration, 3D reconstruction, image registration, motion estimation,...)
- **Image compression**: finding the smallest bit rate while keeping the best quality
- **Signal transmission**: maximizing spectral efficiency as a function of channel propagation conditions
- **Wireless communications**: Bandwidth allocation
- **Image segmentation** by functional optimization to find contours, homogeneous areas, (e.g. Mumford Shah functional)
- **Classification** (SVM, Random Forest, Neural Networks,...)
- **Fusion** of multimodal data
- **Blind source separation**
- ...

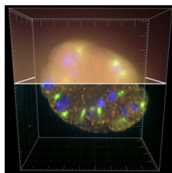
Opimization in GdR IASIS

Not only users but contributors

Typical problems: Inverse problems

Estimate variable u from **noisy incomplete observed data** g

- Restoration/Deconvolution/super-resolution



g : observation

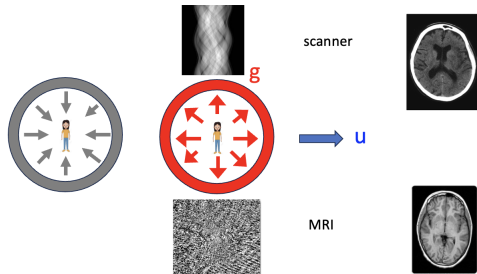
u : image we want to retrieve

Biological Imaging, Satellite Imaging, Astrophysical Imaging

Typical problems: Inverse problems

Estimate variable u from **noisy incomplete observed data** g

- Image reconstruction (ex: medical imaging)



Typical problems: Inverse problems

Estimate variable \mathbf{u} from **noisy incomplete observed data** \mathbf{g} through the physical observation system

$$\mathbf{g} = \mathbf{A}\mathbf{u} + \mathbf{n}$$

$\mathbf{g} \in \mathbb{R}^m$ observed noisy degraded data,

$\mathbf{u} \in \mathbb{R}^n$ reconstructed/restored super-resolved image,

\mathbf{A} observation matrix (in $\mathbb{R}^m \times \mathbb{R}^n$)

\mathbf{n} multidimensional random variable, additive white Gaussian noise.

Typical problems: Inverse problems

Estimate variable \mathbf{u} from **noisy incomplete observed data** \mathbf{g} through the physical observation system

$$\mathbf{g} = A\mathbf{u} + \mathbf{n}$$

$\mathbf{g} \in \mathbb{R}^m$ observed noisy degraded data,

$\mathbf{u} \in \mathbb{R}^n$ reconstructed/restored super-resolved image,

A observation matrix (in $\mathbb{R}^m \times \mathbb{R}^n$)

\mathbf{n} multidimensional random variable, additive white Gaussian noise.

Regularized least square solution

$$\hat{\mathbf{u}} = \arg \min_{\mathbf{u} \in \mathbb{R}^N} \left\{ \frac{1}{2} \|A\mathbf{u} - \mathbf{g}\|_2^2 + R(\mathbf{u}) \right\}$$

Typical problems

$$\hat{\mathbf{u}} = \arg \min_{\mathbf{u} \in \mathbb{R}^N} \{ \|A\mathbf{u} - d\|_2^2 + R(\mathbf{u}) \}$$

- $R(\mathbf{u}) = \|\mathbf{u}\|_2^2, \|\mathbf{u}\|_p^p, \|\mathbf{u}\|_1, \|\mathbf{u}\|_0, \|D\mathbf{u}\|_2^2, \|D\mathbf{u}\|_p^p, \|D\mathbf{u}\|_1, \dots$
- Non differentiability of $\|\cdot\|_1$, non convexity of $\|\cdot\|_p^p, 0 < p < 1$, non continuity of $\|\cdot\|_0$ (NP-hard problem),...

Typical problems

$$\hat{\mathbf{u}} = \arg \min_{\mathbf{u} \in \mathbb{R}^N} \{ \|A\mathbf{u} - d\|_2^2 + R(\mathbf{u}) \}$$

- $R(\mathbf{u}) = \|\mathbf{u}\|_2^2, \|\mathbf{u}\|_p^p, \|\mathbf{u}\|_1, \|\mathbf{u}\|_0, \|D\mathbf{u}\|_2^2, \|D\mathbf{u}\|_p^p, \|D\mathbf{u}\|_1, \dots$
- Non differentiability of $\|\cdot\|_1$, non convexity of $\|\cdot\|_p^p, 0 < p < 1$, non continuity of $\|\cdot\|_0$ (NP-hard problem),...
- **Algorithms** from gradient descent type, proximal, primal-dual, alternating directions method of multipliers, Branch-and-bound,...

Typical problems

$$\hat{\mathbf{u}} = \arg \min_{\mathbf{u} \in \mathbb{R}^N} \{ \|\mathbf{A}\mathbf{u} - \mathbf{d}\|_2^2 + R(\mathbf{u}) \}$$

- $R(\mathbf{u}) = \|\mathbf{u}\|_2^2, \|\mathbf{u}\|_p^p, \|\mathbf{u}\|_1, \|\mathbf{u}\|_0, \|\mathbf{D}\mathbf{u}\|_2^2, \|\mathbf{D}\mathbf{u}\|_p^p, \|\mathbf{D}\mathbf{u}\|_1, \dots$
- Non differentiability of $\|\cdot\|_1$, non convexity of $\|\cdot\|_p^p, 0 < p < 1$, non continuity of $\|\cdot\|_0$ (NP-hard problem),...
- **Algorithms** from gradient descent type, proximal, primal-dual, alternating directions method of multipliers, Branch-and-bound,...
- **Stochastic interpretation**: Maximum a posteriori using Bayes rule

$$\hat{\mathbf{u}} = \arg \max_{\mathbf{u} \in \mathbb{R}^N} P(\mathbf{g} | \mathbf{u}) \cdot P(\mathbf{u})$$

- Stochastic optimization algorithms, SGD, SA, sampling from multidimensional density,...

Parameter estimation

$$J(\mathbf{u}, \theta) = \frac{1}{2} \|A(\theta_1)\mathbf{u} - \mathbf{g}\|_2^2 + R(\mathbf{u}, \theta_2)$$

- **Deterministic approaches**

- Regularizing parameter estimation: Cross-validation, L-curve, homotopy, bi-level method, NN learning,
- Joint estimation with constraints on parameters, e.g. physical constraints on θ_1 ,
- ...

Parameter estimation

$$J(\mathbf{u}, \theta) = \frac{1}{2} \|A(\theta_1)\mathbf{u} - \mathbf{g}\|_2^2 + R(\mathbf{u}, \theta_2)$$

- **Deterministic approaches**

- Regularizing parameter estimation: Cross-validation, L-curve, homotopy, bi-level method, NN learning,
- Joint estimation with constraints on parameters, e.g. physical constraints on θ_1 ,
- ...

- **Stochastic approaches**

- Stein Unbiased Risk Estimator (SURE),
- Maximum Likelihood (ML) estimator with problem of sampling on n-dimensional Gibbs distribution,
- Expectation-minimization (EM) algorithm for ML estimator with latent variables,
- ...

Discrete/Continuous Sparse Optimization

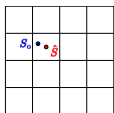


Figure: Discrete case

- the reconstructed peaks are necessarily on the fine grid;
- (non-)convex combinatorial optimization;
- a lot of algorithms;
- large literature.

$$\arg \min_{u \in \mathbb{R}^N} \frac{1}{2} \|g - Au\|_2^2 + \lambda \|u\|_0 \text{ or } 1$$

Discrete/Continuous Sparse Optimization

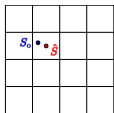


Figure: Discrete case

- the reconstructed peaks are necessarily on the fine grid;
- (non-)convex combinatorial optimization;
- a lot of algorithms;
- large literature.

$$\arg \min_{u \in \mathbb{R}^N} \frac{1}{2} \|g - Au\|_2^2 + \lambda \|u\|_{0 \text{ or } 1}$$

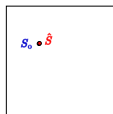


Figure: Off-the-grid case

- not limited by the grid;
- functional convexity on an **infinite dimensional** space;
- measure space with non Hilbertian structure, non reflexive Banach space;
- more recent field of research.

$$\arg \min_{m \in \mathcal{M}(\mathcal{X})} \frac{1}{2} \|g - \mathcal{A}m\|_2^2 + \lambda \|m\|_{TV}$$

Optimization is everywhere in Learning, Signal, Image and Vision!

Thank you for your attention

Parameter estimation: deterministic approach

$$J(\mathbf{u}, \theta) = \frac{1}{2} \|A(\theta_1)\mathbf{u} - \mathbf{g}\|_2^2 + R(\mathbf{u}, \theta_2)$$

- Regularizing parameter estimation: Cross-validation, L-curve, homotopy, bi-level method, NN learning, ...

$$\hat{\theta} = \arg \min_{\theta \in \Theta} \mathcal{L}(\hat{\mathbf{u}}(\theta))$$

$$\hat{\mathbf{u}}(\theta) = \arg \min_{\mathbf{u} \in \mathbb{R}^N} \left\{ \frac{1}{2} \|A(\theta_1)\mathbf{u} - \mathbf{g}\|_2^2 + R(\mathbf{u}, \theta_2) \right\}$$

Parameter estimation: stochastic approach

$$J(\mathbf{u}, \theta) = \|A(\theta_1)\mathbf{u} - \mathbf{g}\|_2^2 + R(\mathbf{u}, \theta_2)$$

- Stein Unbiased Risk Estimator (SURE)
- Maximum Likelihood (ML) estimator with problem of **sampling** on n-dimensional Gibbs distribution
- **Expectation-minimization** (EM) algorithm for ML estimator with latent variables:

E-step: compute $E_{\mathbf{u}|\mathbf{g}, \theta^k} [\log P(\mathbf{g}, \mathbf{u}|\theta)] = Q(\theta, \theta^k)$

M-step: $\theta^{k+1} = \arg \max_{\theta \in \Theta} Q(\theta, \theta^k)$